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Rivet on
Decimal Arithmatic

AN

ATTEMPT

TO ILLUSTRATE THE

USEFULNESS

OF

DECIMAL ARITHMETIC.

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TO ILLUSTRATE THE 2/

USEFULNESS

OF

DECIMAL ARITHMETIC,

IN THE

Rev[d]. Mr. *Brown*'s METHOD

Of WORKING

INTERMINATE FRACTIONS.

By WILLIAM RIVET, Esq.

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AN
ATTEMPT
TO ILLUSTRATE THE
USEFULNESS
OF
DECIMAL ARITHMETIC.

THE most conſiderable Improvements in Decimal Arithmetic, that have been made ſince the Days of Mr. COCKER and Mr. WINGATE, (two of the beſt Authors in the laſt Century) conſiſt in the Manner of working Interminate Fractions; and of reducing our Coin to Decimal Numbers, and valuing thoſe Numbers at Sight.

And it may be doubted, if any Thing new has been produced, that was not exhibited before the Revolution, by the Ingenious and Reverend Mr. BROWN, (who invented the *Rotula)* at leaſt with equal Perſpicuity?

I had the Honour to be one of his intimate Acquaintance, and learned more by a few Hours Converſation with him, than I could have attained by Years of Study; and ſhall not now preſume to offer any Thing of my own, but will only communicate what I collected from his Inſtructions.

To begin with the Reduction of Money. All the Difficulties therein are owing to the Subdiviſions of a Pound Sterling: For if the Integer contained exactly 1000 Parts as a Mill Ree (or Quarter of a Moi d'Or) in *Portugal* doth, our Arithmetic would be all in round Numbers.

But

But having only 960 real Parts in a Pound, we are obliged to ſupply the Deficiency by the Addition of 40, and then ſuppoſe the Pound to conſiſt of 1000 Farthings; whereof every 50 is accounted one Shilling, 25 are reckoned ſix Pence, and all under 24 are ſo many real Farthings.

By theſe Proportions we can at Sight reduce any decimal Number to Money; all Figures on the left Hand of the Pointer being Pounds, and of thoſe to the right Hand, only the firſt three Figures (or the thouſandth Parts) are the Fractions of a Pound.

Hence alſo we are inſtructed how to expreſs Money in decimal Fractions; for we muſt ſet down fifty thouſandth Parts for every Shilling, and for any Thing leſs than a Shilling as many Thouſandths as

there be real Farthings, adding one if they amount to or exceed twenty-four.

But ſuch Numbers only as are aliquot Parts of 100, will terminate rightly; for unleſs the two laſt Figures of the thouſandth Parts are Cyphers, or 25, or 50, or 75, they will be imperfect, and require more Places in the Decimal. To ſupply which Defect, multiply the Difference or Exceſs of the Thouſandths by .04 (which is the decimal Fraction of $\frac{1}{25}$th) adding 1 to every 24 in the Product, and you gain the two ſubſequent Figures in the Place of hundred thouſandths. And if the fourth and fifth Figures be imperfect, proceed in the ſame Manner to gain two more Places in the Decimal, which will then be exact, or end in a ſimple Interminate, which is tantamount.

These

These Rules will be more evident by the following

EXAMPLES.

Let 2*s.* 18*d.* and 1*s.* be expressed in Decimal Numbers.

For the 2*s.* we set down $\frac{100}{1000}$ Parts, (which is $\frac{50}{1000}$ for each Shilling) expressed thus - - - - - - - .100
For the 18*d.* $\frac{50}{1000}$ and $\frac{25}{1000}$ thus .075
For the 1*s.* only $\frac{50}{1000}$ thus - .050
And 6*d.* being the Half of 1*s.* is .025

These four Numbers are perfect in the Place of Thousandths, an Unit being added in each Decimal, for every 24 Farthings in the Sum expressed.

Now, if 6*d.* or 24 Farthings, require the Addition of an Unit, the Decimal of every single Farthing must have the $\frac{1}{24}$th

Part of that Unit added thereto to make it complete.

Thus for one Farthing we ſet down .001 thouſandth, but the Decimal is imperfect, for it is not a full 24^{th} Part of the true Decimal of ſix Pence, but wants the Addition of $\frac{1}{24}^{th}$ of 1, which is .0416, and if this Number be ſubjoined thus .0010416, it will perfect the Decimal of a Farthing.

For Proof of which, divide the Decimal of 6*d.* by 24, and the Quotient will be .0010416.

Here obſerve the Reaſon of ſubſtituting .04 inſtead of .0416, as a Coefficient for perfecting the Decimals. A Twenty-fifth being leſs than a Twenty-fourth only by 1 in 24; and the leſſer Number being more eaſy to work by in the Head; for if you multiply by .04, and add 1 to every

24

24 in the Product, it will be the ſame as if you had multiplied by .0416.

Some Fractions of Money will be perfect at the fourth or fifth Place in the Decimal, as the following,

4*d.* $\frac{1}{2}$ 3*d.* and 1*d.* $\frac{1}{2}$

Four Pence Halfpenny contain 18 Farthings, for which ſet down $\frac{18}{1000}$ thus .018, to perfect which Number multiply the 18 by .04, and the Product is 72, wherein are contained three 24*s.* ſo that 3 added to the 72 compleats the Decimal, thus .01875.

For three Pence ſet down .012, then multiply the 12 by .04, which produces 48, and add 2 for the two 24*s.* therein, and the Decimal is perfect, *viz.* .01250.

One Penny Halfpenny is .006, and .04 times 6 make 24, to which add 1, and the Decimal is .00625.

For

For Proof of the laſt two Numbers, divide the Decimal of ſix Pence by 2, and the Quotient by 2, thus

2) .025

2) .0125 the Decimal of 3*d.*

.00625 ditto of three Halfpence.

Other Numbers require ſix or ſeven Places in the Decimal, as

11*d.* $\frac{1}{4}$ 6*d.* $\frac{3}{4}$ and 5*d.* $\frac{1}{4}$

For eleven Pence Farthing ſet down .046, which is one more than the real Number of Farthings; and as 46 exceeds 25 (which is a perfect Number) by 21, multiply 21 by .04, it will produce 84, to which add 3, and the hundred thouſandths will be 87. But this is imperfect by an Exceſs of 12 above 75, wherefore multiply 12 by .04, adding 1 to each 24 in the Product, and the two laſt Figures will

will be 50, so the perfect Decimal of 11*d*.$\frac{1}{4}$ is .0468750.

Six Pence three Farthings are .028 thousandths, which exceed 25 by 3, and .04 times 3 make 12, and .04 times 12, adding as before directed, make 50, which compleats the Number thus .0281250.

Five Pence one Farthing is .021, and .04 times 21 produce 84, and 3 added make 87, which being 12 above 75, that Excess of 12 multiplied by .04 produces 50, as in the last Example, and the Decimal will be .0218750.

All the foregoing Examples are of perfect Numbers that terminate in 5 or in a Cypher; but there are other Fractions of Money that will be interminate, as

$\frac{1}{3}^{d}$, $\frac{1}{6}^{th}$, $\frac{1}{12}^{th}$, and $\frac{1}{24}^{th}$ Part of 6*d*.

A Third of 6*d*. is 2*d*. which in the Thousandths is expressed thus .008. Now

.04

.04 times 8 is 32, and 1 added makes 33 in the fourth and fifth Places; but 33 exceeds 25 by 8, and the ſame Exceſs multiplied by .04 will produce 33 again, ſo the Fraction will be interminate, having a continued Series of Threes, which is tantamount to a perfect Number, its Deficiency being only 1 in 10.

A Penny, or $\frac{1}{6}$th Part of 6*d.* is expreſſed thus .004, and .04 times 4 make 16 for the hundred thouſandths, and .04 times 16 will produce 64, and 2 added will be 66, to go in the ſixth and ſeventh Places of the Decimal; but 66 exceeds 50 by 16, and that multiplied again by .04, will give the ſame Product continually, wherefore the Decimal is interminate, to wit .0041666.

An Halfpenny, or $\frac{1}{12}$th Part of 6*d.* is in the Thouſandths .002, multiply 2 by .04, it produces .08, and .04 times 8 is

32,

32, and 1 being added, (becauſe above 24) make 33 for the two laſt places, and need no lower reduction, ſo the Decimal is .0020833.

A Farthing, or $\frac{1}{24}$th Part of 6*d.* is .001 in the Thouſandths, .04 times 1 is .04 in the Place of hundred thouſandths, and .04 times 4 are 16, which is interminate, and the Decimal ſtands thus .0010416.

Theſe few Examples ſhew all the Variety that can happen in the Reduction of Engliſh Money to decimal Fractions, and hereby we obtain a ſure Rule to know when any given Decimal is adequate to the Sum expreſſed by the thouſandth Parts; for if .04 times the Exceſs of the Thouſandths is the ſame with the two ſubſequent Figures, and .04 times the Exceſs of thoſe, the ſame with the ſixth and ſeventh Figures, the Decimal is perfect.

Note,

Note, the continuing the Decimals of Money lower than thouſandth Parts, is never neceſſary but when ſome farther operation is to be performed therewith, as multiplying, dividing, adding, or ſubtracting them.

The Manner of working ſuch decimal Fractions as are interminate, comes next to be conſidered.

And here it may be of Uſe to ſhew, how vulgar Fractions are converted into Decimals, and how to reduce them back into vulgar.

To reduce a vulgar Fraction to a Decimal one of equal Value, ſet down the Denominator for a Diviſor, and the Numerator for a Dividend, adding thereto as many Cyphers as you want Places in the Quotient; fix a Pointer between the Numerator and the Cyphers, and work as in a common Diviſion of whole Numbers.

EXAM-

EXAMPLES.

Let $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{25}$ be reduced.

$$4)\ \underline{1.00} \qquad 8)\ \underline{1.000} \qquad 25)\ \underline{1.0\overset{\times}{0}}\ (.04$$

$$.25 \qquad\qquad .125 \qquad\qquad \underline{100}$$

These Quotients (if continued) would terminate in Cyphers, and are therefore perfect Numbers.

Let $\frac{1}{6}$, $\frac{1}{12}$, and $\frac{1}{24}$ be reduced.

$$6)\ \underline{1.000} \qquad\qquad 12)\ \underline{1.0000}$$

$$.1\cancel{6} \qquad\qquad .0833$$

$$\begin{array}{l} 24)\ 1.0\overset{\times}{0}00\ (.041\cancel{6} \\ \qquad\ \ \underline{96} \\ \qquad\ \ \ 40 \\ \qquad\ \ \ \underline{24} \\ \qquad\ \ \ 160 \\ \qquad\ \ \ \underline{144} \\ \qquad\ \ \ \ 16 \end{array}$$

The

The three laſt Quotients would continually repeat the ſame Figure, and are therefore called ſimple Interminates. But there are other Fractions, which in the Reduction of them to Decimals, do not repeat a ſingle Figure only, like the foregoing, but after producing a few Figures in the Quotient, the Remainder will be the ſame Number you began with, and then of Courſe you will have a Repetition of the ſame Figures over again, and theſe are called compound Interminates.

Of which Sort are $\frac{1}{7}$, $\frac{1}{13}$, and $\frac{1}{73}$.

$$7)\ 1.000000000000$$

$$.142857142857+1$$

```
13) 1.00 (.076923+1
     91
     --
     90
     78
     --
     120
     117
     ---
      30
      26
      --
       40
       39
       --
        1
```

```
73) 1.00 (.01369863+1
     73
     --
     270
     219
     ---
      510
      438
      ---
       720
       657
       ---
        630
        584
        ---
         460
         438
         ---
          220
          219
          ---
            1
```

When

When theſe compound Fractions conſiſt of an even Number of Figures in the Decimal or Circle they form, they moſt commonly have this Property, that one Half of the Circle added to the other Half, will always produce a Series of Nines; wherefore the Denominators of compound Interminates, are different from thoſe of perfect Decimals, and of ſimple Interminates.

For Inſtance, the Decimal of $\frac{1}{7}$ contains a Circle of ſix Figures, to wit, .142857 *&c.*

Now the firſt and fourth Figures, the ſecond and fifth, and the third and ſixth (if added together) will make Nines, begin at what Part of the Circle you pleaſe.

$$\begin{array}{r} 142 \\ 857 \\ \hline 999 \end{array} \qquad \begin{array}{r} 285 \\ 714 \\ \hline 999 \end{array} \qquad \begin{array}{r} 571 \\ 428 \\ \hline 999 \end{array}$$

So the Decimal of $\frac{1}{73}$ is a Circle of eight Figures, *viz.* .01369863 *&c.*

Begin

Begin wherever you will, if the firſt four Figures be added to the four ſubſequent ones, the Total will be always a Series of Nines; as for Example,

$$\begin{array}{rr} 1369 & 6986 \\ 8630 & 3013 \\ \hline 9999 & 9999 \end{array}$$

which peculiar Property muſt always be regarded when any of theſe Fractions are to be reduced to their loweſt Terms; for as the Denominators of perfect Decimals and ſimple Interminates, are an Unit with Cyphers ſubjoined, ſo the Denominators of compound Interminates are always as many Nines as there are Places in the decimal Fraction.

Any Decimal may be reduced to a vulgar Fraction, by ſubſcribing its Denominator; for Inſtance,

.3125 $.83\dot{3}$ and .65853 *&c.*

The firſt is a perfect Number, the ſecond a ſimple Interminate, denoted by the Daſh through the laſt Figure, and the third is a compound Interminate, expreſſed by the *&c.* which ſignifies the ſame Figures would be repeated continually.

Subſcribe their Denominators, and they become vulgar Fractions, thus

$$\frac{3125}{10000} \qquad \frac{833}{1000} \qquad \frac{65853}{99999}$$

Theſe and the like Fractions are reduced to their loweſt Terms by finding a common Meaſure in the ordinary Way of reducing vulgar Fractions; though generally perfect Numbers may be ſooner reduced by a continued Diviſion by 5.

$$\begin{array}{l} .3125)\ 1.0000\ (3 \\ \qquad\qquad \underline{9375} \\ \qquad\qquad 0625)\ 3125\ (5 \\ \qquad\qquad\qquad\qquad \underline{3125} \\ \qquad\qquad\qquad\qquad\quad 0 \end{array}$$

The

The common Meaſure is .0625, by which divide the Numerator and Denominator, and they will produce a new Fraction in its loweſt Terms.

```
.0625) 3125 (5, new Numerator
       3125
       ----
          0

          ×
.0625) 1.0000 (16, new Denominator
         625
         ----
         3750
         3750
         ----
            0
```

The ſecond Reduction.

$5)\ \frac{3125}{10000} \mid \frac{652}{2000} \mid \frac{125}{400} \mid \frac{25}{80} \mid \frac{5}{16}$ths.

By each Way of reducing it, the perfect Decimal .3125 appears to be equal to ^{5}

Reduce $\frac{833}{1000}$ to its loweſt Terms.

```
833) 1000 (1
     833
     ----
     .166) 833 (5, Numerator
           833
           ---
            00
```

The common Meaſure is .166

```
166) 1000 (6, Denominator
     1000
     ----
       00
```

Anſwer, $\frac{5}{6}$ are the loweſt Terms.

Reduce $\frac{65853}{99999}$ to its loweſt Terms.

```
65853) 99999 (1
       65853
       -----
       34146) 65853 (1
              34146
              -----
              31707) 34146 (1
                     31707
                     -----
The common Meaſure   .02439) 31707 (13
                             2439
                             ----
                              7317
                              7317
                              ----
                               000
```

.02439)

```
.02439) 65853 (27, Numerator
        4878
        -----
        17073
        17073
        -----
           00

.02439) 99999 (41, Denominator
        9756
        -----
        02439
        02439
        -----
           00
```

The Anſwer is $\frac{27}{41}$ the loweſt Terms.

Reduce $\frac{714285}{999999}$ to its loweſt Terms.

```
714285) 999999 (1
        714285
        ------
        285714) 714285 (2
                571428
                ------
Common Meaſure .142857) 285714 (2
                        285714
                        ------
```

142857) $\frac{714285}{999999}$ ($\frac{5}{7}$ the loweſt Terms.

As interminate Fractions, both ſimple and compound, have certain Deficiencies peculiar to themſelves, the Manner of adding and ſubſtracting them muſt differ from that of perfect Numbers; but in all of them Care muſt be taken to ſet Tenths, Hundredths, and Thouſand Parts *&c.* exactly under each other, and to ſeparate from the Total as many decimal Places as are contained in the loweſt Fraction added or ſubſtracted.

If ſimple Interminates are to be added, all the Sums muſt be continued to an equal Number of Places; and in caſting up the Units, you are to carry by Nine (inſtead of Ten) to ſupply the Want of the ſubſequent Figures, that would have been expreſſed, had they continued lower; for, as in the Reduction of theſe Numbers to Decimals there is always a Remainder of one in ten, ſo in caſting up or multiplying the

the Units, if you carry only Nines (inſtead of Tens) it amounts to the ſame as taking in that Remainder.

And if perfect Decimals are mixed with Interminates, the latter muſt be continued, at leaſt one Place, lower than the former, otherwiſe their Deficiency cannot ſo well be ſupplied.

Let the Decimals of 13*s.* 4*d.* 11*s.* 8*d.* and 5*s.* 5*d.* be added; alſo the Decimals of 8*s.* 4*d.* 6*s.* 10*d.* and 4*s.* 8*d.*

l. *s.* *d.*		*s.* *d.*	
13 : 4	.6666$\cancel{6}$	8 : 4	.4166$\cancel{6}$
11 : 8	.58333	6 : 10	.34166
5 : 5	.27083	4 : 8	.23333
1 : 10 : 5	1.5208$\cancel{3}$	19 : 10	.9916$\cancel{6}$

The Units in the firſt Example amount to 12, wherefore I ſet down 3 and carry one; in the ſecond Example they amount to 15, which is 6 above 9, wherefore I

put 6 in the Unit's Place, by which Means the Deficiency is ſupplied.

If the Decimals of 13*s.* 4*d.* 6*s.* 8*d.* and 3*s.* 4*d.* are to be added, two Places in each Number will be ſufficient.

Theſe Decimals, expreſſed as low as thouſandths, ſtand thus $.66\cancel{6}$ $.33\cancel{3}$ and $.16\cancel{6}$

	l.	*s.*	*d.*
$.6\cancel{6}$		13 :	4
$.3\cancel{3}$		6 :	8
$.1\cancel{6}$		3 :	4
$1.1\cancel{6}$	1 :	3 :	4

By this Example it appears, that ſimple Interminates may often be wrought with more Eaſe and fewer Figures than many perfect Numbers.

EXAMPLES

Of perfect Decimals and ſimple Interminates added together, *viz.* 14*s.* 2*d.*

17*s.*

17*s.* 9*d.* and 8*s.* 10*d.*; alſo 15*s.* 4*d.* 16*s.* 3*d.* and 17*s.* 7*d.*

l.	*s.*	*d.*	
	14 :	2	.70833
	17 :	9	.8875
	8 :	10	.44166
2 :	0 :	9	2.03750

l.	*s.*	*d.*	
	15 :	4	.766666
	16 :	3	.81250
	17 :	7	.879166
2 :	9 :	2	2.458333

Compound Interminates are likewiſe to be continued to an equal Number of Places, and the full Extent of their Circles, before they are caſt up; and their Deficiency is ſupplied by trying what the ſubſequent Figures would have brought, had the Circles been repeated, which you may always know, if they conſiſt of but few Places,

Places, as do the Decimals of $\frac{1}{13}$ $\frac{1}{41}$ and $\frac{1}{365}$

Let $\frac{11}{13}$ and $\frac{8}{13}$ and $\frac{5}{13}$ be added; alſo $\frac{9}{13}$ and $\frac{6}{13}$ and $\frac{4}{13}$

.846153 &c.	.692307 &c.
.615384	.461538
.384615	.307692
1.846153	1.461538
Total 1 and $\frac{11}{13}$	Total 1 and $\frac{6}{13}$

.571428 &c.	.714285 &c.
.428571	.857142
.285714	.142857
1.285714	1.714285
Total 1 and $\frac{2}{7}$	Total 1 and $\frac{5}{7}$

In theſe Examples the Circles extend as low as Millionths, and then repeat; ſo that the Figures in the Place of Tenths would follow, if the Fractions were continued lower: But that is unneceſſary, becauſe if you

you caſt up the Tenths firſt in your Mind, you know how many you are to carry to the Millionth Parts, in order to ſupply their Deficiency; and this is a more certain Way than carrying by Nines, as in the Caſe of ſimple Interminates, though in general that Method will anſwer, as appears by theſe four Examples.

When circular Numbers extend to a very great Length before they repeat, it is much eaſier to perform the Work in vulgar Fractions, and then reduce your Product or Total to a decimal Fraction; there being many Numbers that will contain above fifty or threeſcore Figures, and ſome almoſt an hundred Figures in the Quotient, before they begin to repeat, as the Decimals of $\frac{1}{97}$ $\frac{1}{61}$ and $\frac{1}{59}$ to all which Numbers the Rule of *Approximation* is generally applied, which is this, *viz.*

Make

Make two Operations; firſt with the given Numbers which are leſs than Perfection; then with the ſame Numbers, increaſing the loweſt Figure by one, which will be more than the Truth, and ſo far as the two Products or Totals agree, the Decimal muſt be perfect. See p. 39.

But we proceed to other Examples.

Let $\frac{25}{41}$ and $\frac{13}{41}$ and $\frac{10}{41}$ be added; alſo $\frac{90}{365}$ and $\frac{70}{365}$ and $\frac{50}{365}$

.60975 *&c.*	.24657534 *&c.*
.31707	.19178082
.24390	.13698630
1.17073	.57534246
Total 1 and $\frac{7}{41}$	Total $\frac{210}{365}$

Let $\frac{35}{73}$ and $\frac{26}{73}$ and $\frac{12}{73}$ be added.

.47945205 *&c.*	.05479452 *&c.*
.35616438	.38356164
.16438356	.56164383
.99999999	1.00000000

The

The Total of the firſt Sum is a Series of Nines, which is equal to Unity; wherefore in caſting up the ſecond Sum, I carried by Nine (inſtead of Ten) in the Units Place, which ſupplied the Deficiency; and if one had been added to the Units in the firſt Sum, it would have had the ſame Effect.

SUBSTRACTION requires the like Diſpoſition of ſetting Tenths under Tenths, Hundredths under Hundredths, &c.; and as many Places muſt be ſeparated in the Difference, as there be in the loweſt of the given Numbers.

Interminates (whether ſimple or compound) muſt be of an equal Length; and always continued lower than perfect Numbers when both are given.

In ſubſtracting Infinites (or ſimple Interminates) from Infinites, or from perfect Numbers,

Numbers, you are to borrow but Nine (instead of Ten) in the Unit's Place; and in compound Interminates you have Regard to what the subsequent Figures would produce, if the Fractions were continued lower.

EXAMPLES.

4.375	10.4375	$1.3\dot{3}$
3.650	$8.333\dot{3}$	$.6\dot{6}$
.725	$2.1041\dot{6}$	$.6\dot{6}$

$.666\dot{6}$	.428571 &c.	.846153 &c.
.525	.285714	.538461
$.141\dot{6}$	.142857	.307692

MULTIPLICATION of decimal Fractions is the same as of whole Numbers; and so are all other Operations therein.

The general Rule is to separate as many decimal Places in the Product, as are contained

tained in both Factors; and if the Product doth not contain ſo many Figures, ſupply the Deficiency by placing Cyphers on the left Hand, and then fix the Pointer or Decimal Note.

EXAMPLES.

35.2	.125	25.375
4.5	.25	4.75
1760	625	126875
1408	250	177625
158.40	.03125	101500
		120.53125

If either of the Factors be an interminate Fraction, make it the Multiplicand, and if it be a ſimple Interminate, carry by Nines in the Multiplication of the Units, and when the Work is performed, make every Line of an equal Length before you caſt them up, as was directed in the Rule of Addition.

EXAM-

EXAMPLES.

$25.3\cancel{3}$	$.16\cancel{6}$	$.158\cancel{3}$
2.75	$4\cancel{3}.5$	$245.$
$1266\cancel{6}$	$83\cancel{3}$	$791\cancel{6}$
$1773\cancel{3}3$	500	$633\cancel{3}3$
$506\cancel{6}66$	$66\cancel{6}66$	$316\cancel{6}66$
$69.666\cancel{6}$	7.2500	$38.791\cancel{6}$

Let 4578 Dollars, at 3*s.* 4*d.* per Dollar, be reduced to Pounds Sterling; the Decimal of 3*s.* 4*d.* is $.166\cancel{6}$.

$.166\cancel{6}$	$.166\cancel{6}$	$.1\cancel{6}$
$4578.$	$4578.$	$4578.$
13333	13333	133
$11666\cancel{6}$	11666	$11\cancel{6}$
$8333\cancel{3}3$	8333	$8\cancel{3}3$
$6666\cancel{6}66$	6666	$66\cancel{6}66$
763.0000	762.9293	763.00

Here the ſame Queſtion is wrought three different Ways, whereby it appears, that conſiderable Loſs would be ſuſtained, if the

the ſeveral Products were not continued to an equal Length, before they were caſt up; for in the ſecond Operation, where the Figures that ſhould be on the right Hand of the diagonal Line, are not ſupplied, the Total proves very defective.

And the laſt Operation manifeſtly ſhews, that ſimple Interminates require fewer Figures to work with than more perfect Numbers frequently do.

When the Multiplicand is a compound Interminate, let it be continued to the full Extent of the Circle, that you may try what would be brought to the Figure in the Unit's Place; and let every Line of the Products be continued equally low, before you caſt them up, taking Care that the Figures ſo ſupplied are ſuch as properly belong to the reſpective Circles.

Let $\frac{5}{7}$ and $\frac{90}{365}$ be multiplied by 423.

.714285 *&c.*	.24657534 *&c.*
423.	423.
2.142857	73972602
14285714	493150684
28571428$\cancel{5}$	98630136$\cancel{9}$8
302.142857	104.30136986
Product 382 $\frac{1}{7}$	Product 104 $\frac{110}{365}$

Four Men are concerned in a joint Undertaking, which originally belonged to thirteen Partners, whoſe Capital was divided into as many Shares, but now their Proportions are as follow, viz. *A* is poſſeſſed of $\frac{1}{13}$, *B* has $\frac{3}{13}$, *C* has $\frac{4}{13}$, and *D* $\frac{5}{13}$. They are to make a Dividend between them of 780*l.* Profits of the laſt three Months Trade; query each Man's Proportion thereof?

Take the Decimal of $\frac{1}{13}$ (to wit) .076923, *&c.* for a common Factor, which multiply

multiply by the Sum that is to be divided, and the Product will be exactly one thirteenth Part of the Sum.

		Factor	
A's Share	60 *l.*	.076923 *&c.*	
B's ——	180 *l.*	780.	the Sum,
C's ——	240 *l.*	———	
D's ——	300 *l.*	615384$\cancel{6}$	
	——	538461$\cancel{5}$3	
	780 *l.*	———	
	——	60.000000	

Examples of decimal Fractions that will not circulate unless they be continued to a great Number of Places, in multiplying of which we usually make two Operations, as before has been observed under the Head of Approximation.

Let $\frac{50}{67}$ be multiplied by 432.

.7462686 −	.7462687 +
432.	432.
14925372	14925374
22388058	22388061
29850744	29850748
322.3880352	322.3880784

The two Products agree in the fractional Parts as far as to ten thousandths, which is sufficient, in case no further Operation is to be made therewith.

Multiplying by 10, 100, 1000, or any other Decuple, is performed by removing the Pointer forward, as many Places as you have Cyphers in the Multiplier.

Thus, the Decimal of 8 *s.* 5 *d.* $\frac{1}{4}$, which is .4218750, if multiplied

by 10	would be	4.218750
by 100	———	42.18750
by 1000	———	421.8750
by 10000	———	4218.750

All

All which may be performed in the Mind, without tranſcribing the Figures.

But in Diviſion the Work is quite the Reverſe, by removing the Pointer back towards the left Hand, juſt ſo many Places as there are Cyphers in the Diviſor. For as carrying the Pointer forward, gives the ſeveral Decuples of any Number, ſo bringing it back, ſhews the reſpective Decimals of the ſame Number.

Diviſion, in other Reſpects, is the ſame as in whole Numbers; in which Care muſt be taken in preparing the Dividend, and ſettling the Quotient.

The Dividend muſt contain as many decimal Places, as the Diviſor and Quotient together; and the Quotient muſt have exactly ſo many as the Dividend hath more than the Diviſor.

If it happens that the Quotient doth not contain ſo many Figures, the Defect

muſt be ſupplied by prefixing one or more Cyphers, ſo as to take Place in the Decimal; and if the Dividend be a whole Number, ſubjoin a ſufficient Quantity of Cyphers, that you may have Places enough in the Quotient.

Laſtly, if the Dividend contains exactly the ſame Number of decimal Places as are in the Diviſor, the Quotient will be a whole Number.

In 216*l.* Sterling, how many Pezzos, at 54*d.* per Pezzo.

The Decimal of one Pezzo is .225

```
.225) 216.000 (960 Pezzos
      2025
      ----
       1350
       1350
       ----
       0000
```

In

In 279*l.* how many Mill Rees, at 5*s.* 7*d.* ½ per Mill Rez?

The Decimal of 5*s.* 7*d.* ½ is .28125.

```
.28125) 279.00000 (992. Mill Rees
        253125
        ------
         258750
         253125     4) 992
         ------        ---
           56250       248. Moi D'ors
           56250
           -----
           0000
```

In 609*l.* 8*s.* 3*d.* how many Livres, at 45*d.* per Ecus, or Crown of 3 Livres, or 60 Sous Tournois?

One Livre is .0625

```
.0625) 609.41250 (9750.6
       5625
       ----
        4691
        4375
        ----
         3162
         3125
         ----
           3750
           3750
           ----
             00
```

Anſwer 9750 Livres and 12 Sous.

If either or both of the Factors be Interminate, work by the following Rules.

When the *Dividend* is an interminate Number, and you have Occasion to continue it lower, it muſt be done with the proper Figures belonging to the ſame Fraction, whether ſimple or compound.

When the *Diviſor* is an Interminate, the firſt Product thereof, when placed under the Dividend, muſt be continued to an equal Length therewith, and when ſubſtracted, it will produce a new Dividend; and the ſame Method muſt be repeated as often as any Figure is continued in the Quotient.

If a perfect Number is to be divided by an interminate one, let the Dividend end in a Cypher, elſe the firſt Subſtraction may prove wrong.

In 133*l.* 1*s.* 4*d.* $\frac{1}{2}$, how many Dollars, at 37*d.* $\frac{3}{4}$?

```
.157291$\not 6$) 133.0687500 (846
          1258333333
          ----------
           72354166
           62916666
           --------
            9437500
            9437500
            -------
            000000
```

Answer 846. Dollars.

Let $\frac{10}{37}$ be divided by $\frac{1}{16}$ in decimal Numbers.

```
.0625) .270270270 (4.32432 &c.
        25 0
        ----
         2027
         1875
         ----
          1520
          1250
          ----
           2720
           2500
           ----
            2027
            1875
            ----
             1520
             1250
             ----
              270
```

The Quotient is $4\frac{12}{37}$

Let

Let $\frac{1}{4}$ of $\frac{1}{13}$ be divided by $2\frac{1}{2}$ decimally, that is .019230769 *&c.* by 2.5

$$
\begin{array}{l}
2.5)\ .019230769\ (.00769230 \\
\quad\ \ \underline{175} \\
\quad\quad\ 173 \\
\quad\quad\ \underline{150} \\
\quad\quad\quad 230 \\
\quad\quad\quad \underline{225} \\
\quad\quad\quad\quad 57 \\
\quad\quad\quad\quad \underline{50} \\
\quad\quad\quad\quad\ 76 \\
\quad\quad\quad\quad\ \underline{75} \\
\quad\quad\quad\quad\quad 19
\end{array}
$$

The Quotient is $\frac{1}{130}$ Part, or $\frac{1}{10}$ of $\frac{1}{13}$

Let

Let 8 and $\frac{4}{7}$ be divided by $\frac{1}{3}$ and $\frac{2}{3}$ds

```
·33) 8.5714285 (25.71428
     66666666
     --------
     19047619
     16666666
     --------
      2380952          8.57
      2333333             3
      -------          ----
        47619          25.71 = 25 5/7
        33333
        -----
        14285
        13333
        -----
          952
          666
          ---
          285
          266
          ---
         1904 &c.
```

66)

$.6\cancel{6})\ 8.5714285\ (12.85714$
$6\cancel{6}666666$

19047619 $\qquad 8.57$
$13\cancel{3}33333$ $\qquad 1\frac{1}{2}$

5714285 $\qquad 8.57$
$53\cancel{3}3333$ $\qquad 4.28$

380952 $\qquad 12.85 = 12\frac{6}{7}$
$33\cancel{3}333$

47619
$46\cancel{6}66$

952
$6\cancel{6}6$ $\qquad 2)\ 25.71428$

285 $\qquad 12.85714$ $\Big\}$ Proof.
$26\cancel{6}$

1904 &c.

It is presumed the Examples before given under each Rule, are sufficient to explain the Manner of working interminate Fractions, unto any one who is tolerably acquainted with decimal Arithmetic;

tic; and to the conſideration of ſuch perſons only theſe ſheets are ſubmitted.

One peculiar Advantage of round Numbers is, that all Reduction from higher to lower Denominations, and back again from a lower to a ſuperior Denomination, is entirely ſaved, every Sum being at once expreſſed both in its higheſt and loweſt Denomination, the Pointer, or decimal Note, fixing its Value, and the Removal thereof increaſing or diminiſhing the ſame, in a tenfold Proportion; which is the Reaſon of their being conſidered as whole Numbers in all Operations.

This Property of being multiplied, or divided, only by removing the Pointer forward or backward, and thereby obtaining the ſeveral Decuples, or Decimals, of any given Sum, or Quantity, makes theſe Numbers the fitteſt of any for conſtructing all Sorts of Tables. Nine Lines only, and

thoſe

thoſe containing but few Figures, being ſufficient to form a Table of very great Extent; whereas Tables conſtructed in vulgar Numbers, beſides being liable to various typographical Miſtakes, require more than ten thouſand times the Number of Figures to anſwer the ſame Purpoſes; of which an Example ſhall be given in one general Table of Intereſt for Days.

The Income for one Day of 1 *l.* per Annum, is .002739726 *&c.* which is the Quotient of 1 *l.* divided by 365.

A Table conſtructed from this Quotient, muſt neceſſarily ſhew, by Inſpection, the daily Income of any annual Sum, and conſequently will be applicable to Queſtions of Intereſt for Days.

A TABLE

at 100 l. *per Cent. per Annum.*

Indices.	Tabular Sums.
1	.002739726
2	.005479452
3	.008219178
4	.010958904
5	.013698630
6	.016438356
7	.019178082
8	.021917808
9	.024657534

The Indices denote the yearly Revenue, and the tabular Sums give the Income thereof for one Day.

	Thousandths.		*l.*	*s.*	*d.*	
Thus 6*l. per A.* yields *per D.*	.016	or	0	0	$3\frac{3}{4}$	78
60 - - - - - -	.164	-	0	3	$3\frac{1}{4}$	81
600 - - - - - -	1.643	-	1	12	$10\frac{1}{2}$	08
6000 - - - - - -	16.438	-	16	8	9	82.

Increase

Increaſe the annual Sum by any Number of Days, and the Table will equally ſhew the Income for thoſe Days.

If the Income of the laſt mentioned yearly Sums was required for ten Days on each Sum, the Anſwers would be as follows:

	Thouſandths.
The 6 *l. per Ann.* would produce	.164
60 - - - - - - - -	1.643
600 - - - - - - - -	16.438
6000 - - - - - - - -	164.383

Every Sum being increaſed ten times.

If a Widow's Jointure of 500 *l. per Ann.* is in Arrear fourſcore Days, what is due for that Time?

	Index	Tab. Number
500 *per Ann.*		
80 Days	40000 —	109.589 *&c.*
40000		

The Anſwer is 109 *l.* 11 *s.* 9 *d.* $\frac{1}{4}$ and $\frac{48}{100}$ of a Farthing.

To

To apply this Table to Intereſt for Days, requires only the finding one Year's Intereſt of the principal Sum, which is always an hundredth Part of the Product of the ſame Sum, multiplied by the Rate *per Cent.*

Thus the Intereſt for one Year, of 2150*l.* principal Money, at the Rate of 4*l. per Cent. per Ann.* is found to be 86*l.* and if you would know ten Days Intereſt of the ſame Principal, take the tabular Sums anſwering to 860. in the Indices, which is increaſing the yearly Sum by the Time, as before directed.

2150. Principal	}	An hundredth Part of the Product is 86*l.* one Year's Intereſt.
4. the Rate		
8600. Product		

	Indices		Tabular Sums
86 *per Ann.*	800.	—	2.19178
10 Days	60.	—	.16438
860			2.35616

Anſwer, 2*l.* 7*s.* 1*d.* ¼ 92 the Intereſt for 10 Days.

Other Examples.

685 at 5 *per Cent.* for 26 Days

685	
5	
34.25	one Year's Intereſt
26	
205.50	
6850	
890.50	

800.	—	2.191780
90.	—	.246575
0.5	—	.001369
		2.439726

Anſwer, 2 *l.* 8 *s.* 9 *d.* $\frac{1}{2}$ 14 hundredth of a Farthing.

2500*l.* at 3 $\frac{1}{2}$ for 68 Days

2500	
3 $\frac{1}{2}$	
7500	
1250	
87.50	a Year's Int.
68	
70000	
52500	
5950.00	

Indices		Tabular Sums
5000	—	13.69863
900	—	2.46565
50	—	.13698
		16.30136

12000*l.* at 4 $\frac{1}{4}$ for 80 Days

4 $\frac{1}{4}$			
48000	40000	—	109.58904
3000	800	—	2.19178
510.00 *per Ann.*			111.78082
80.			
40800.			

Anſwer, 111 *l.* 15 *s.* 7 *d.* $\frac{1}{4}$ 59

In the ſame Manner, Tables may be raiſed for almoſt every Purpoſe, only by expreſſing the Value of an Unit in a decimal Fraction, and multiplying the ſame through the nine Figures.

There are certain Officers in the Navy, whoſe Pay is 13 *s.* 6 *d. per Diem*; now if one Day's Wages be reduced to a Decimal, and a Table formed thereon, the Value of any Number of Days Pay may be known by Inſpection, which may be of great Uſe to a Pay-Clerk in the Hurry of Buſineſs. The Decimal of one Day's Pay is .675.

The TABLE is as follows:

Days.	Wages.
1	0.675
2	1.350
3	2.025
4	2.700
5	3.375
6	4.050
7	4.725
8	5.400
9	6.075

What is due to a Captain of a third Rate for 134 Days?

100 Days —	67.5
30 ——	20.25
4 ——	2.7
	90.45

The Anſwer is 90 *l.* 9 *s.*

The ſeveral Parts of Time reduced to decimal Numbers, will be found very ſerviceable in the Conſtruction of aſtronomical Tables, &c. as they greatly facilitate and ſhorten the Calculations.

The Decimal of 1 Hour (or $\frac{1}{24}$ of 1 Day) is .0416; $\frac{1}{6}$ of $\frac{1}{10}$ of 1 Hour gives the Decimal of 1 Minute, *viz.* .0006944

and

and $\frac{1}{6}$ of $\frac{1}{10}$ of 1 Minute is the Decimal of 1 Second, $.00001157\dot{4}0\dot{7}$ &c.

$$
\begin{array}{l}
24)\ 1.0\dot{0}00\ (.041\dot{6} \text{ the Decimal of 1 Hour} \\
\underline{\ 96\ } \\
\ \ 40 \\
\ \ \underline{24} \\
\ \ 160 \\
\ \ \underline{144} \\
\ \ \ 16
\end{array}
$$

$6)\ \underline{.00416\dot{6}}$ one Tenth of 1 Hour

$.00069\dot{4}$ the Decimal of 1 Minute

$6)\ \underline{.000069444\dot{4}}$ one Tenth of 1 Minute

$.00001157\dot{4}0\dot{7}$ Decimal of 1 Second

In like Manner the Decimals of 3^ds^, 4^ths^, 5^ths^, &c. are produced, and may be formed into Tables by multiplying them through the nine Figures, as before directed.

 The

The Exceſs of the mean tropical ſolar Year, above the Egyptian Year of 365 Days, amounts to 5 Hours and 49 Minutes.

The Decimal of	5 h.	0 min.	—	.208333
of		40	— —	.027777
of		9	— —	.006250
				.242361

And as every Equinox and Solſtice falls 5 h. 49 min. later than in the preceeding Year, a Table formed from this Number .242361 may be applied to many uſeful Purpoſes.

For Inſtance, In 1753 the Sun entered Libra, in a Meridian that lies 156° Weſt of Greenwich, on September the 22^{d} (or 265^{th} Day from the Calends of January) preciſely at Noon, and 10 h. 24 min. later at Greenwich.

Now if we would know the Time of the autumnal Equinox in the preſent Year 1763,

1763, we muſt add to the Time in 1753, as many times 5 h. 49 min. as Years have elapſed, rejecting the Days, and the Anſwer is given:

The Interval between 1753 and 1763 is ten Years, the Decimal of 5 h. 49 min. anſwering to one Year is .242361; therefore ten Years will be 2.42361, of which the two Days are to be rejected, and only the Decimal .42361 retained, which is equal to 10 h. 10 min.

The Reduction		.4236
		24
		16944
		84722
	H.	10|166
		60
	M.	10|000

	D.	H.	M.
Sun in Libra 1753, Sept.	22	0	0
Add for 10 Year's Interval		10	10
	22	10	10
Add for Meridian of Greenwich		10	24
In 1763, Sept.	22	20	34

It would be endleſs to enumerate all the Advantages of theſe Numbers, whereby we can raiſe Factors for whatever Uſes we may have Occaſion.

For Inſtance, in Caſes of Exchange;

Reduce an Unit of the foreign Money to a Decimal of a Pound Sterling, and it becomes a common Factor; which, if multiplied by any Quantity of the foreign Money, gives the Value of the whole in Engliſh.

And if any Sum of Engliſh Money be divided by the ſame Factor, the Quotient will be its Value in the foreign Money.

A

A Bill is drawn for 54 Mill Rees, at 67*d.* $\frac{1}{2}$ per Mill Ree.

$$\begin{array}{lrrrr} \text{The Decimal of 1 Mill} & & & & \\ \quad\text{Ree is - - -} & .28125 & & & \\ & 54 & & & \\ \hline & 112500 & & & \\ & 140625 & & & \\ \hline & & l. & s. & d. \\ & 15.18750 = & 15 & 3 & 9 \end{array}$$

Let 236*l.* 5*s.* Sterling be reduced to Mill Rees, of 67*d.* $\frac{1}{2}$ per Mill Ree.

$$\begin{array}{rlll} .28125) & 236.25000 & (840 \text{ Mill Rees} & \\ & 225000 & & \\ \hline & 112500 & & \\ & 112500 & 4)\ 840 & \\ \hline & 00000 & 210 & \text{Moi d'Ors} \end{array}$$

Proof:

Proof: 1.125 one Moi d'Or

$$
\begin{array}{r}
1.125 \\
210 \\
\hline
11250 \\
2250 \\
\hline
236.250
\end{array}
= \begin{array}{ccc} l. & s. & d. \\ 236 & 5 & 0 \end{array}
$$

Here the ſame Factor anſwers both Queſtions.

See other Examples in *Multiplication* and *Diviſion.*

I am engaged in a public Undertaking, wherein my Share or Intereſt is a one and fortieth Part of the Profit and Loſs upon the whole.

Now the Decimal of $\frac{1}{41}$ is .02439 &c. And if this Factor be multiplied by any given Sum, the Product will be a one and fortieth Part of that Sum, whereby I may ſurely know what I am to pay or receive. See the Caſe in page 38 and 39.

If

If I am to pay any particular Duty, as ſuppoſe the old Subſidy on Tobacco, which is 5*l.* per Cent. upon the Rate for every Pound of Tobacco, with a Diſcount of 25*l.* per Cent. for prompt Payment;

The Decimal of the Duty or Subſidy is .0031250; multiply this Factor by the Number of Pounds of Tobacco, and the Product will be the Sum to be paid.

The Wages of an able Seaman at 22*s.* per Month, is per Diem $.039\overset{*}{2}85714\overset{*}{2}$ &c.

Let this Factor be multiplied by the Number of Days, and the Wages are ſeen by the Product.

The Clerks in Chancery charge 10*d.* per Sheet for Copies of Bills, Anſwers, and other Pleadings.

The Decimal of 10*d.* is $.041\cancel{6}$

Which Factor, if multiplied by any Number

ber of Sheets, produces the Sum to be paid to the Clerk.

To form a Chronological Table that will shew the Day, Hour, and Minute, of the Autumnal Equinox in any Year of the World.

Reduce the Quantity of a tropical Year to a decimal Number, which will be 365.24236111; multiply this Factor through the nine Figures, and the Table is formed.

1	365.242361
2	730.484722
3	1095.727083
4	1460.969444
5	1826.211805
6	2191.454166
7	2556.696527
8	2921.938888
9	3287.181250

Q.

Q. The Time of the Sun's entering Libra A. M. 3000, the firſt Year after Biſſextile?

Years		Days
3000	——	1095727.083 = to 20 d. or 2 h.

7) 1095727

156532 + 3 ⊙

	D.	H.	M.
Epoch for Libra	297	0	0
3000 Yrs. Interval		2	0
	297	2	0
Retroceſſion	22		
	275	2	0
	273		
Sunday, October	2	2	0
		10	24
October	2	12	24 at Greenw.

The

The ſame Table converted into vulgar Numbers.

Years.	D.	H.	M.	Years.	D.	H.	M.
9000	3287181	6	0	90	32871	19	30
8000	2921938	21	20	80	29219	9	20
7000	2556696	12	40	70	25566	23	10
6000	2191454	4	0	60	21914	13	0
5000	1826211	19	20	50	18262	2	50
4000	1460969	10	40	40	14609	16	40
3000	1095727	2	0	30	10957	6	30
2000	730484	17	20	20	7304	20	20
1000	365242	8	40	10	3652	10	10
900	328718	3	0	9	3287	4	21
800	292193	21	20	8	2921	22	32
700	255669	15	40	7	2556	16	43
600	219145	10	0	6	2191	10	54
500	182621	4	20	5	1826	5	5
400	146096	22	40	4	1460	23	16
300	109572	17	0	3	1095	17	27
200	73048	11	20	2	730	11	38
100	36524	5	40	1	365	5	49

Q. The tropical Reduction of 1440 Years?

1000	——	365242	8	40
400	——	146096	22	40
40	——	14609	16	40
	Anſwer	525949 Days.		

I might

I might tire the Reader by inſtancing one Tythe of the Caſes wherein theſe Numbers excel our common Arithmetic; but hope I have produced enough to ſhew their Superiority; wherefore I ſhall leave the further Application of them to thoſe whoſe Buſineſs may require it, and who have Leiſure to purſue the agreeable Taſk.

I freely confeſs, that, for my own Part, I never had Occaſion for greater Skill in Numbers, than would enable me to caſt up a Tradeſman's Bill; but having ſubſcribed for Mr. BROWN's *Arithmetica Infinita*, on its firſt Publication in 1718, I was deſirous to know on what Principle his Tables were formed, and the Pleaſure attending my Inquiry, engaged me in the Study of this Species of Arithmetic, ſo far as to acquire that little Knowledge I have

have therein; and I ſhall think my Trouble amply recompenced, if what is now imparted may be of Uſe to Society.

FINIS.